ICPC Thailand National Competition 2024

- Tutorial Slide -

8th September 2024

A- Auntie's Magical Cake

- Problem Author: Akarapon Watcharapalakorn
- Solved by 20 teams.
- First solved after 20 minutes.

A- Auntie's Magical Cake

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- Consider eating the i-th cake, where $1 < i < N/2$, and let the increases in deliciousness from the left-side and right-side cakes be x and y, respectively.
- If you change the position from i to $i+1$, the new increases in deliciousness will be $x+i$ and $y-(i+1)$. It's clear that the total in the first case, $x+y$, is always higher than in the second case, $x+i+y-(i+1) = x+y-1$.

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- If you change the position from i to $i + 1$, the new increases in deliciousness will be $x + i$ and $y - (i + 1)$. It's clear that the total in the first case, $x + y$, is always higher than in the second case, $x + i + y - (i + 1) = x + y - 1$.
- The same method can also be applied for cases where $i > N/2$.

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- Can be optimized to $O(1)$ with math.

B- Back in the Day

- Problem Author: Mattanyu Tangngekkee
- Solved by 34 teams.
- First solved after 14 minutes.

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- For each part, greedily choose the biggest character possible. Then put the smallest character remained in the front. For example 2222 becomes *ac* and 7777777777 becomes *qss*.

B- Back in the Day

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- **Be careful**, each number contains different amount of characters. '7' and '9' contains 4 characters each while the rest contains just 3.
- Time complexity is $O(|S|)$.

C- Cattering

- Problem Author: Natapong Sriwatanasakdi
- Solved by 1 teams.
- First solved after 44 minutes.

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- We can determine whether all cats can have happiness value at least h by **maximum bipartite matching**.
- We build a bipartite graph of cats and foods. There exist an edge between cat i and food j if and only if $A_{ij} \geq h$. All cats can have happiness value at least h if and only if the **maximum bipartite matching size** of this graph is N.

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- In this problem, we can use **binary search** to find the maximum possible value of the minimum happiness for all cats.
- We can determine whether all cats can have happiness value at least h by **maximum bipartite matching**.
- We build a bipartite graph of cats and foods. There exist an edge between cat i and food j if and only if $A_{ij} \geq h$. All cats can have happiness value at least h if and only if the **maximum bipartite matching size** of this graph is N.
- Therefore, we can binary search to find the maximum value of happiness that all cats can reach. The search is $O(\log M)$ times since there are NM possible values—values in matrix A.
- Time complexity is $O(Matching \cdot \log M)$.

C- Cattering

- The official solution used Hopcroft-Karp Algorithm for matching.
- Kuhn Algorithm with modifications also works. In each vertex, try finding an available vertex that it can reach first before DFS on matched vertices. The algorithm can be further improved by shuffling edges first.
- One of the problem testers also uses Dinic's Algorithm. While Dinic's Algorithm has the same time complexity as Hopcroft-Karp Algorithm in case of maximum bipartite matching, the overhead from creating network flow is larger, hence the actual execution time is significantly slower.

D- Disinfection Patch

- Problem Author: Attitarn Buathep
- Solved by 3 teams.
- First solved after 111 minutes.

D- Disinfection Patch

- Suppose that $min_x, max_x, min_y, max_y$ are the minimum and the maximum value of x and y coordinates for the drops of disinfection respectively, we can construct a rectangle that cover all of the drops which have (min_x, min_y) and (max_x, max_y) as the bottom-left and the top-right corner of the shape.

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- We can do the same for the bacteria, then compare the ratio between the length of each side of the two rectangles.
- If the ratio are not equal to each other, output -1. Otherwise, we must scale the disinfection rectangle with the ratio and shift the corners to the bacteria rectangle.

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- There will be some special cases such as when x or y are the same for all points, so the length of some side of the rectangle becomes 0.
- Time complexity is $O(N \log N)$

E- Executive's Holidays

- Problem Author: Mattanyu Tangngekkee
- Solved by 0 teams.

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- For the i-th day, if there requires a_i executives for the meeting, then it also means there can be at most $N - a_i$ executives unattended that day.

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  - For example, let the quota for executives unattended each day are $[1, 2, 0, 2, 1, 3, 2]$.
  - The first executive can only take a break on the day that has the quota. Therefore he can at most take a break from day 4 to day 7. After that, the quota becomes $[1, 2, 0, 1, 0, 2, 1]$.
  - The next executive can take a break from day 1 to day 2, and the quota left will be $[0, 1, 0, 1, 0, 2, 1]$ and so on...

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- The rest of the unattended quota can be distributed however since it would not increase the sum.

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- Finding the longest non-zero subarray for an array takes $O(T)$. Removing one for every member in the subarray also takes $O(T)$.
- Since you have to repeat the process N times, time complexity is $O(N \cdot T)$, which is not fast enough.

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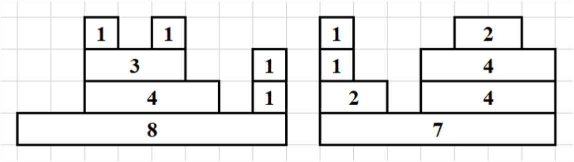

Figure: The histogram and its partition of the array $[1, 1, 4, 3, 4, 2, 1, 3, 0, 4, 2, 1, 3, 4, 4, 3]$

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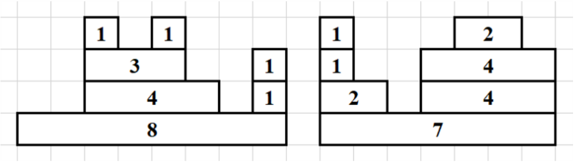

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- When you sort the partition by its length from longest, it is the same range as the longest non-zero subarray previously, so the answer is the sum of the first N partition's length.

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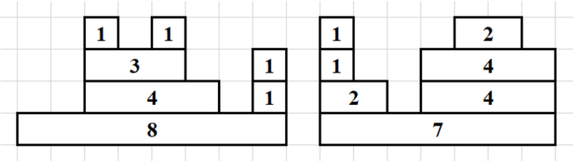

Figure: The histogram and its partition of the array $[1, 1, 4, 3, 4, 2, 1, 3, 0, 4, 2, 1, 3, 4, 4, 3]$

- When you sort the partition by its length from longest, it is the same range as the longest non-zero subarray previously, so the answer is the sum of the first N partition's length.
- You can obtain such partitions by iterating from left to right and keep the position and the height in a stack.

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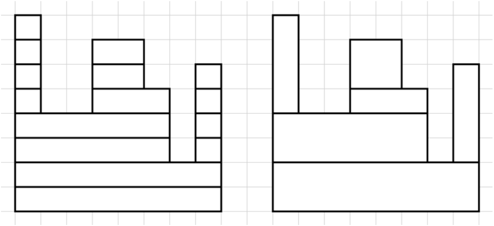

Figure: Example of an optimized partition.

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- Time complexity is $O(T)$

F- Fill T

- Problem Author: Mattanyu Tangngekkee
- Solved by 0 teams.

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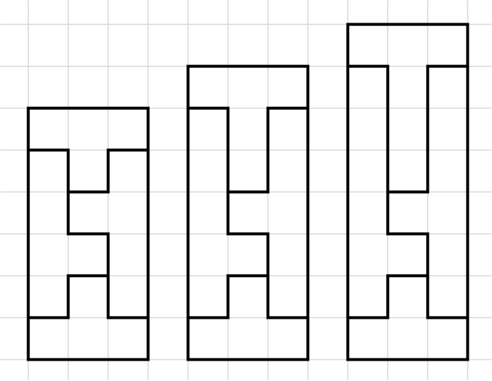
- If $R = 1$ or $R = 2$, it is obvious that the grid cannot be filled.
- For $R = 3$ and $C \leq 5$, it also can be shown that the grid cannot be filled.
- That is all the grids that cannot be filled.

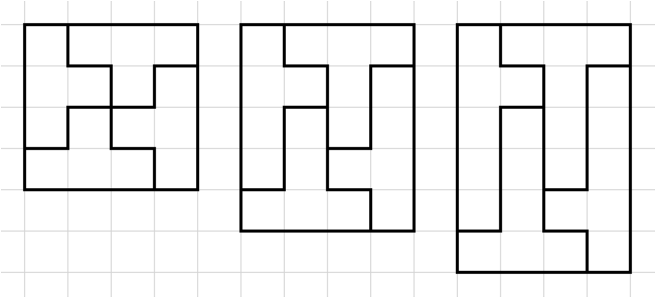
F- Fill T

- For $R = 3$ and $C \geq 6$, the grid can be filled with the pattern below.



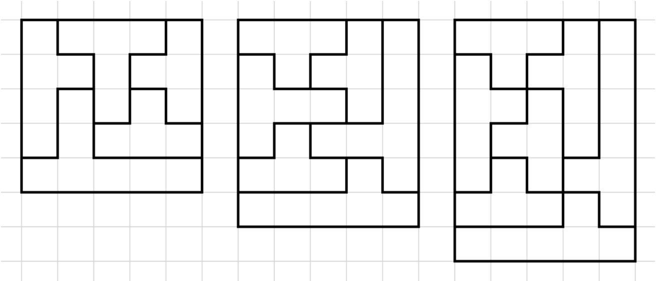
F- Fill T

- For $R = 4$, the grid can be filled with the pattern below.



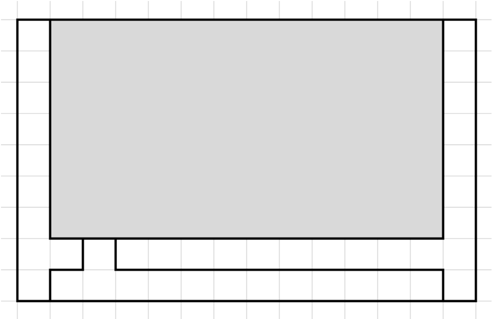
F- Fill T

- For $R = 5$ and $C = 5$, 6, or 7, the grid can be filled with the pattern below.



F- Fill T

- For $R = 5$ and $C \geq 8$ or $R \geq 6$, you can use 3 pieces of T-shape to form the border so that you can recursively fill out the $(R-2) \times (C-2)$ grid with the pattern below.



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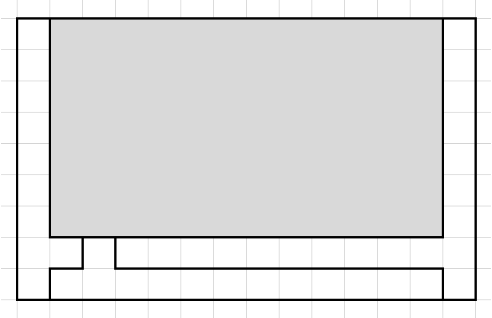

- Time complexity is $O(\max(R, C))$

G- Glory Road

- Problem Author: Mattanyu Tangngekkee
- Solved by 45 teams.
- First solved after 12 minutes.

G- Glory Road

- Let the vertices of the triangle be (A_x, A_y), (B_x, B_y) and (C_x, C_y) and the midpoint of each side be (P_x, P_y), (Q_x, Q_y) and (R_x, R_y).
- You can write the system equation as follows:

$$\begin{cases} \dfrac{A_x + B_x}{2} = P_x \\ \dfrac{B_x + C_x}{2} = Q_x \\ \dfrac{A_x + C_x}{2} = R_x \end{cases} \qquad \begin{cases} \dfrac{A_y + B_y}{2} = P_y \\ \dfrac{B_y + C_y}{2} = Q_y \\ \dfrac{A_y + C_y}{2} = R_y \end{cases}$$

G- Glory Road

- The system equation has a solution of:

$$\begin{cases} A_x = P_x - Q_x + R_x \\ B_x = P_x + Q_x - R_x \\ C_x = -P_x + Q_x + R_x \end{cases} \qquad \begin{cases} A_y = P_y - Q_y + R_y \\ B_y = P_y + Q_y - R_y \\ C_y = -P_y + Q_y + R_y \end{cases}$$

- Just be careful about the order of the input and the output.
- Time complexity is $O(1)$.

H- Heavenly Sequence

- Problem Author: Poonyapat Sriroth
- Solved by 0 teams.

H- Heavenly Sequence

- First, it is obvious that we will only choose the maximum X (by choosing $X = R$).

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- A sequence is considered **Heavenly** if and only if every permutation of every subsequence is **Good**.
- In other words, a sequence is **Heavenly** if, when we pick **any numbers in any order** from the sequence to create a new sequence, that new sequence is also **Good**.

H- Heavenly Sequence

- By using the rearrangement inequality (or through mathematical observation), we can simplify the definition of **Heavenly** to:

Lemma 1

A sequence $b[1], b[2], b[3], \ldots, b[n]$ is **Heavenly** if and only if, after sorting the sequence, for each $i \leq j$, the subsequence $b[i], b[i+1], b[i+2], \ldots, b[j]$ is **Good**.

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- This lemma can be proven by fixing the maximum and minimum of the new sequence and then maximizing the right-hand side of the inequality.
  - This is achieved by selecting every value between the maximum and minimum and sorting that sequence.
  - Considering all possible maximum and minimum values and combine everything will results in above observation.

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- Suppose the sorted sequence is $b[1], b[2], b[3], \ldots, b[n]$. Given the previous observation, there are still many **Good** sequences to consider.

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- However, we can reduce this by noting that:

Lemma 2

If $b[1], b[2], b[3], \ldots, b[j]$ is **Good**, then for any i with $i \leq j$, the subsequence $b[i], b[i+1], b[i+2], \ldots, b[j]$ is also **Good**.

H- Heavenly Sequence

- Thus, we only need to consider K such that for every j (where $j \leq N$), $b[1], b[2], b[3], \ldots, b[j]$ remains **Good**.

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- Thus, we only need to consider K such that for every j (where $j \leq N$), $b[1], b[2], b[3], \ldots, b[j]$ remains **Good**.
- To achieve this, we need to find K that satisfies the inequality (we know that the minimum is $b[1]$ and maximum is $b[j]$):

$$K \cdot b[1] \geq b[j]^2 + \sum_{i=1}^{j-1} b[i] \cdot b[i+1] - X \cdot b[j]$$

for $j = 2, 3, \ldots, N$.

H- Heavenly Sequence

- Consequently, we seek a data structure that can efficiently solve the right side of the inequality to find the minimum K:

$$K \cdot b[1] \geq \max \left(b[j]^2 + \sum_{i=1}^{j-1} b[i] \cdot b[i+1] - X \cdot b[j] \right)$$

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for $j = 2, 3, \ldots, N$.

- The right side of this inequality requires us to find the maximum value of linear equations at specific points. Since this operation must be performed online (both inserting lines and retrieving answers), one suitable data structure for this is the **lazy Li Chao tree**. You can learn more about it at `https://codeforces.com/blog/entry/86731`

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- Time complexity is $O(N \log^2 N)$

I- Ideal Permutation Pairing

- Problem Author: Mattanyu Tangngekkee
- Solved by 4 teams.
- First solved after 81 minutes.

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- Let P is a permutation size N have a rank of p if it is the p-th smallest permutation of size N. For shorten, we can say that $rank(P) = p$.

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Lemma

Let $P = p_1 p_2 p_3 \dots p_N$ and $Q = q_1 q_2 q_3 \dots q_N$ be permutations of size N. If $rank(Q) = rank(P) + t \cdot (N-k)!$ for some k and t, then $p_{k+1} p_{k+2} \dots p_N$ and $q_{k+1} q_{k+2} \dots q_n$ have the same ordering.

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Proof

We can prove by induction from $k = N \to 1$. The $k = N$ part is obvious. If $F(k)$ is true, considering the possible value of p_k if $p_1 p_2 \dots p_{k-1}$ is fixed, there are $N - k + 1$ possibles candidates, which also is the list $[p_k, p_{k+1}, \dots, p_n]$. That means if $rank(Q) = rank(P) + t \cdot (N-k)! = rank(P) + t \cdot (N-k-1) \cdot (N-k)!$, the rank of p_k among the list $[p_k, p_{k+1}, \dots, p_n]$ and the rank of q_k among the list $[q_k, q_{k+1}, \dots, q_N]$ is equal, making $p_k p_{k+1} \dots p_N$ and $q_k q_{k+1} \dots q_N$ have the same ordering, thus $F(k-1)$ is true.

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- Since there are $N!$ permutations size N in total, if P and Q forms an ideal pair and P is smaller than Q, then $rank(Q) = rank(P) + \frac{N!}{2}$.

I- Ideal Permutation Pairing

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- Since there are $N!$ permutations size N in total, if P and Q forms an ideal pair and P is smaller than Q, then $rank(Q) = rank(P) + \frac{N!}{2}$.
- Because $\frac{N!}{2}$ can be written as $\frac{N \cdot (N-1)}{2} \cdot (N-2)!$. Thus, $p_3 p_4 \dots p_n$ and $q_3 q_4 \dots q_N$ have the same ordering.

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- If p_1, p_2 has the rank of x, then:
  - $x = (p_1 - 1) \cdot (N - 1) + p_2$ if $p_1 < p_2$.
  - $x = (p_1 - 1) \cdot (N - 1) + (p_2 - 1)1$ if $p_1 > p_2$.

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- The rank of q_1, q_2 is $x' = x + \frac{N \cdot (N-1)}{2} \mod N \cdot (N - 1)$, which, conversely, can be used to find the value of q_1, q_2 by:
  - $q_1 = 1 + \lfloor \frac{x'-1}{N-1} \rfloor$
  - $q_2' = 1 + (x' - 1) \mod (N - 1)$
    - $q_2 = q_2'$ if $q_1 < q_2'$.
    - $q_2 = q_2' + 1$ if $q_1 \geq q_2'$.

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- For each i, find q_i such that $rank(q_i) = rank(p_i)$, this can be done in $O(1)$ with counting sort.
- Time complexity is $O(N)$.

J- Jewel Collection

- Problem Author: Mattanyu Tangngekkee
- Solved by 0 teams.

J- Jewel Collection

- **This is not a maximum weight edge cover problem.**
- **This is not a FLOW problem either.**
- Still, it is a graph problem.

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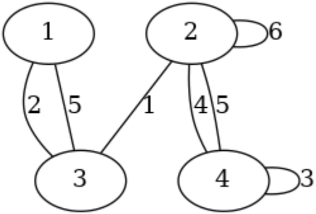

Figure: Graph of the first example.

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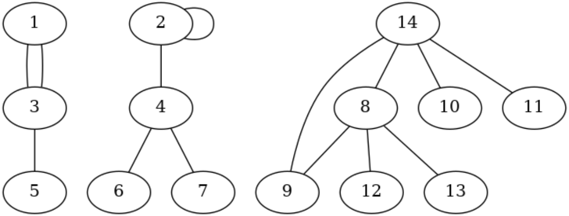

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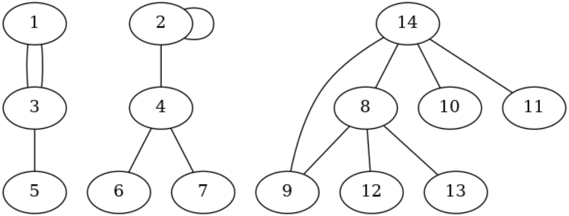

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- Therefore, you need to find a maximal pseudoforest with the maximum sum.

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- To print the matching, just do a simple DFS until you find a cycle.
- Time complexity is $O(N \log N)$

K- Kid Rally

- Problem Author: Attitarn Buathep
- Solved by 2 teams.
- First solved after 186 minutes.

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- If Alice is at (a, b), it can be shown that if she decide to move forward optimally, her new X coordinate will increase by exactly 1. Same goes for Bob.
- Generally speaking, until one of them stops moving, Alice's and Bob's move will increase one X coordinate at a time.
- Moreover, at the same X coordinate, Alice will have to be on the left side of Bob.

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- Right now, time complexity should be $O(MaxValue \cdot N \cdot M)$, but we can reduce to $O(MaxValue \cdot N)$ by observing that there are at most only 10 values per coordinate X that we must consider.

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- There are some special cases. For example, considering this map grid size 2×2:

$$\begin{matrix} 9 & 1 \\ 9 & 0 \end{matrix}$$

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- In this particular grid map, the optimal way is for Alice to stay at $(1, 1)$,earning a score of 9 while Bob moves from $(1, 2)$ to $(2, 1)$, earning a score of 10. Hence, the final score becomes 90.

L- Lulu and Friends

- Problem Author: Supakorn Kijwattanachai
- Solved by 23 teams.
- First solved after 6 minutes.

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- This can be done in $O(|S| + |T|)$ and do it from every *first* matched position will result in $O(|T||S + T|)$ per query.
- Or you can generate all $2^{|T|}$ possible strings from deletion and memorize the answer.

M- Marriage Proposals

- Problem Author: Natapong Sriwatanasakdi
- Solved by 0 teams.

M- Marriage Proposals

- In this problem, the town can be visualized as a tree that have villages as vertices. Each bidirectional road between two villages is an edge.
- Each edge should contain the detail about the shop on the road — it can be described by the pearl type and the purchase limit.

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- For naive solution, We store the price of each pearl type in an array. For every type 2 event, we simply update the corresponding price in the array, which can be done in constant time.

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- For each *type* 1 event, we can perform a breadth-first search (BFS) or depth-first search (DFS) from vertex x to find a path to vertex y.
- Let P be the set of edges along this path. Let e_i be edge i.
- For each pearl type j that appears on the path, total expense is calculated as

$$\sum_{e_i \in P, a_i = j} b_i \cdot c_j$$

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- While this improves the performance, the overall time complexity remains $O(NQ)$.

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- This solution achieves a time complexity of $O(QN^{\frac{2}{3}}\log N)$.

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- We perform DFS on the tree starting from any root vertex. Before starting, we initialize three variables: *turn*, *tout*, and *tour*.
  - *turn* : keeps track of the current DFS step.
  - *tout*[ ] : is an array that records the DFS exit time for each vertex.
  - *tour*[ ] : is a list that tracks the edges traversed during the DFS.

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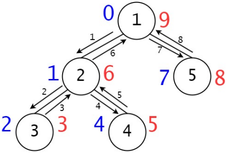
- Similarly, when returning from a child to its parent, we append the edge again and update *tout*[*child*] to the current value of *turn*.



DFS Order: 1→2→3→2→4→2→1→5→1

edges

1	2	3	4
(1,2)	(2,3)	(2,4)	(1,5)

tout

1	2	3	4	5
9	6	3	5	8

tour

1	2	3	4	5	6	7	8
1	2	2	3	3	1	4	4

Figure: DFS traversal of the example tree. Blue numbers represent the entry turn. Red numbers represent the exit turn.

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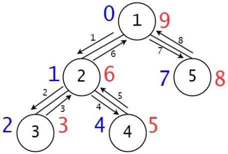
- Let $l = tout[x]$ and $r = tout[y]$. An edge e is part of the path **if and only if** it appears exactly once in the subarray $tour[l \dots (r-1)]$.

M- Marriage Proposals



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- For example, when traveling from vertex 3 to vertex 5, we have $l = 3$ and $r = 8$.
- The path from vertex 3 to vertex 5 is $3 \to 2 \to 1 \to 5$, corresponding to the edges $2 \to 1 \to 4$.
- The subarray $tour[3 \ldots 7]$ is $[2, 3, 3, 1, 4]$.
- The edges that appear exactly once in this subarray are edges 2, 1, and 4, which match the set of edges in the actual path from vertex 3 to vertex 5.

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- For each type 1 event, we query the edges between x and y by examining those that appear exactly once in $tour[l \ldots (r-1)]$, where $l = \min(tout[x], tout[y])$ and $r = \max(tout[x], tout[y])$.

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- In this problem, the total expense cannot exceed 10^7, since the unit prices are capped at 1,000 dollar and the purchase limits for each type of pearl do not exceed 10,000.

M- Marriage Proposals

- The new data structure uses two arrays:
  - *count*[ ] : Tracks the frequency of each expense value.
  - *blockCount*[ ] : Tracks the number of elements in blocks of size $\lceil\sqrt{\text{MaxValue}}\rceil$, where block b covers a range of values from $b\lceil\sqrt{\text{MaxValue}}\rceil$ to $(b+1)\lceil\sqrt{\text{MaxValue}}\rceil - 1$.

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- When querying the maximum value, we first find the highest non-zero block in $blockCount$.
- After identifying the block, we search through $count$ for the exact maximum value.
- Both the number of blocks and the elements within each block are bounded by $O(\sqrt{\text{MaxValue}})$.
- This allows our algorithm to run in $O(QN^{\frac{2}{3}} + Q\sqrt{\text{MaxValue}})$.

N- [N]ew YoRHa Security

- Problem Author: Nitit Jongsawatsataporn
- Solved by 0 teams.

N- [N]ew YoRHa Security

Lemma 1

For any prime number p if $\gcd(y, p-1) = 1$, then $f(a) = a^y \pmod{p}$ maps $\{1, 2, \ldots, p-1\}$ to $\{1, 2, \ldots, p-1\}$.

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Proof

Suppose not for contradiction. Then, there are $c, d \in \{1, 2, \ldots, p-1\}$ such that $c^y \equiv d^y \pmod{p}$. Let z be a natural number that $yz \equiv 1 \pmod{p}$ (exists by Bézout's identity). Raising both sides by z and using Fermat's little theorem yields

$$c \equiv (c^y)^z \equiv (d^y)^z \equiv d \pmod{p}$$

Thus, a contradiction.

N- [N]ew YoRHa Security

Lemma 2

If x, y are two natural number such that $\gcd(x, p-1) = \gcd(y, p-1)$, then $f(a) = a^x$ and $g(a) = a^y$ has the same image.

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- We can solve the problem from these two lemmas by iterating the divisor of $p-1$ and computing the image of a^d. Then we can multiply their contribution and get the desired result.
- Time complexity $O(pd(p-1)\log p)$

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- For another solution, Since $(\mathbb{Z}_p^\times, \times) \cong (\mathbb{Z}_{p-1}, +)$, the problem reduces to:

Given a natural number n, compute the number of pairs (a, b) such that $0 \leq a, b \leq n-1$ and $ab \equiv k \pmod{p}$.

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- We can solve this easily by dividing the number between $0 \to n-1$ into groups based on their greatest common divisor with $p-1$.
- Elements from the same group will have the same answer, and we can compute from their class instead.

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- If p is the generator, then we can map $(\mathbb{Z}_{p-1}, +)$ to $(\mathbb{Z}_p^\times, \times)$ by $a \mapsto p^a$.
- To find primitive root, you can simply check each element whether it generates the group or not.
- Time complexity is $O(p \log p + d(p-1)^2) = O(p \log p)$.